Comparative performance of nodal-based versus edge-based finite element formulations

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The aim of this work is to compare the computational performance of nodal-based finite element formulations versus edge-based finite element formulations. Edge-based formulations are widely adopted in the EM community and known to be stable (M. Salazar-Palma et al., "Iterative and Self-Adaptive Finite-Elements in Electromagnetic Modeling". Artech House Publishers, 1998. J. M. Jin, "The Finite Element Method in Electromagnetics", John Wiley & Sons, 2002). However, straightforward nodal-based finite element formulations suffer from stability problems. Recently, two nodal based formulations overcoming the mentioned stability problems have been proposed by the authors. Details of the two formulations are given below. The comparison is established between each one of the nodal-based formulations and their edge-based counterparts.

The first nodal approximation is the simplification of the weighted regularized Maxwell equation method explained in (R. Otin. "Regularized Maxwell equations and nodal finite elements for electromagnetic field computations." Electromagnetics, vol. 30, pp. 190-204, 2010). This formulation presents several advantages: It provides spurious-free solutions and well-conditioned matrices without the need of Lagrange multipliers, its integral representation is well-suited for hybridization with integral numerical techniques and the nodal solution of the electromagnetic domain is usually easier to couple in multi-physics problems. On the other hand, this regularized formulation presents a serious drawback: if the electromagnetic field has a singularity in the problem domain, a globally wrong solution is obtained. Fortunately, this disadvantage can be corrected, but, in doing this, the well-conditioning of the matrix can be put in jeopardy. Then, the question we try to solve in this work is if the computational advantages of the proposed nodal formulation compensate its drawbacks when compared with the well-establish edge double-curl finite element formulation.

The second nodal formulation we are going to test is based on the algorithm explained in (S. Badia and R. Codina, "A nodal-based finite element approximation of the Maxwell problem suitable for singular solutions," *submitted*) for the Maxwell operator. This formulation will be adapted to the frequency domain and compared with mixed edge element formulations. The main differences between this new nodal formulation and the one based on weighted regularization is the fact that a Lagrange multiplier is needed in order to enforce the divergence-free constraint at the discrete level but there is no need to define the weighting functions. This last point is important, since it does not require to determine *a priori* were the singularities are, making the method easier to apply and implement. On the other hand, it does not exhibit the problem of integrating numerically the weighted terms and no additional (regularized) boundary conditions are needed. Finally, this method has been easily extended to coefficient jumps in (S. Badia and R. Codina, "A combined nodal continuous-discontinuous finite element formulation for the Maxwell problem," Applied Mathematics and Computation, vol. 218, pp. 4276-4294, 2011) using dG-type terms on the material interfaces.