A Numerical Model for the Search of the Optimum Capacitance in Electromagnetic Metal Forming

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Abstract. Electromagnetic forming (EMF) is a high velocity forming technique that uses electromagnetic forces to shape metallic workpieces. In this work we present a numerical model that is able to compute the current flowing trough the coil, the Lorentz force acting on the workpiece and the optimum capacitance of the EMF process. The main advantage of our approach is that it provides an explicit relation between the capacitance of the capacitor bank and the frequency of the discharge, which is a key parameter in the design of an electromagnetic forming system. This method is computationally efficient because it only requires solving the time-harmonic Maxwell equations for a few frequencies to have completely characterized the EMF system. The approach can be very useful for estimating the order of magnitude of some parameters, for experimentation on modeling conditions, for coil design or for modeling complex geometries. Moreover, it can be easily included in a sequential coupling strategy without the worry of numerical instabilities.

Keywords: Electromagnetic forming, optimum capacitance, optimum frequency, numerical analysis, finite element method

INTRODUCTION

Electromagnetic forming (EMF) is a high velocity forming technique that uses electromagnetic forces to shape metallic workpieces. The process starts when a capacitor bank is discharged through a coil. The transient electric current which flows through the coil generates a time-varying magnetic field around it. The time-varying magnetic field induces electric currents in any nearby conductive material. These induced currents flow in the opposite direction to the primary currents and, therefore, a repulsive force arises between the coil and the conductive material. If this repulsive force is strong enough to stress the workpiece beyond its yield point then it can shape it with the help of a die or a mandrel.

This technique present several advantages: no tool marks are produced on the surfaces of the workpieces, no lubricant is required, improved formability, less wrinkling, controlled springback, reduced number of operations and lower energy cost. But, in order to design sophisticated EMF systems and control their performance, it is necessary to advance in the development of theoretical and numerical models of the EMF process. This is the objective of the present work. More specifically, we focus our attention on the numerical analysis of the electromagnetic part of the EMF process.

In this paper we present a numerical model that characterizes an EMF process in frequency domain. The input data required are the geometry and material properties of the system coil-workpiece and the electrical parameters of the capacitor bank. With these data and the time-harmonic Maxwell's equations we are able to calculate the optimum capacitance, the current flowing trough the coil and the electromagnetic forces acting on the workpiece.

FREQUENCY-DOMAIN ELECTROMAGNETIC MODEL

The electromagnetic model presented in this work starts by solving the time-harmonic Maxwell's equations for a given frequency interval. For each frequency ω we compute the electromagnetic fields $\mathbf{E}(\omega)$ and $\mathbf{H}(\omega)$ inside a volume containing the coil and the workpiece. With $\mathbf{E}(\omega)$ and $\mathbf{H}(\omega)$ we calculate the inductance $L_{cw}(\omega)$ and the resistance $R_{cw}(\omega)$ of the system coil-workpiece. With $L_{cw}(\omega)$ and $R_{cw}(\omega)$ we compute the intensity $I(t)$ flowing through the coil. Finally, with $I(t)$ and the fields $\mathbf{E}(\omega)$ and $\mathbf{H}(\omega)$, we obtain the Lorentz force exerted on the workpiece and an estimation of the optimum capacitance of the EMF process. In the following it is described each step in more detail.

Time-harmonic Maxwell's equations

We start solving numerically the time-harmonic Maxwell's equations inside a volume containing the coil and the workpiece. For that, we employ the in-house code called ERMES, which implements in C++ the finite element formulation explained in [1]. The electromagnetic fields $\mathbf{E}(\omega)$ and $\mathbf{H}(\omega)$ obtained at this step will be used to compute the inductance and the resistance of the system coil-workpiece and the Lorentz force acting on the workpiece.

Although, in theory, we need all the frequencies $\omega \in [0,\infty)$ to fully characterized an EMF system, in practice, we have observed in the literature that the frequencies of interest lay in the interval

$$0.5 < \frac{\delta}{\tau} < 1.5 \tag{1}$$

where τ is the thickness of the workpiece and δ is the skin depth. The skin depth is defined as

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \tag{2}$$

being μ the magnetic permeability of the workpiece and σ its electrical conductivity.

Intensity through the coil

The next step is to obtain the electrical current $I(t)$ flowing through the coil. We assume that the coil, workpiece and capacitor bank form a RLC circuit with resistance $R = R_{cb} + R_{cw}$, inductance $L = L_{cb} + L_{cw}$ and capacitance $C = C_{cb}$, where the subscript cb denotes the parameters of capacitor bank and the subscript cw denotes the parameters of system coil-workpiece. The parameters R_{cb}, L_{cb} and C_{cb} are given data, while R_{cw} and L_{cw} are obtained from [2]

$$L_{cw}(\omega) = \frac{1}{|I_n|^2}\int_{\upsilon} \mu|\mathbf{H}(\omega)|^2\, d\upsilon, \tag{3}$$

$$R_{cw}(\omega) = \frac{1}{|I_n|^2}\int_{\upsilon} \sigma|\mathbf{E}(\omega)|^2\, d\upsilon \tag{4}$$

where υ is a volume containing the coil and the workpiece and I_n is the current injected into the system through the input terminals. We can imagine the coil and the workpiece inside the volume υ with only its input terminals protruding and consider the set as a generic, two-terminal, linear, passive electromagnetic system operating at low frequencies. The current $I(t)$ in the RLC circuit formed by the capacitor bank and the system coil-workpiece is given by the expression

$$I(t) = \frac{V}{\omega_0 L}\exp(-\gamma t)\sin(\omega_0 t) \tag{5}$$

being

$$\omega_0 = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} \tag{6}$$

and

$$\gamma = \frac{R}{2L}. \tag{7}$$

The inductance L and the resistance R are evaluated at the frequency ω_0. The parameter V in (5) is a given data that represents the value of the initial voltage at the terminals the capacitor bank. V is related with the energy of the discharge U by

$$U = \frac{1}{2}CV^2. \tag{8}$$

Equation (5) shows that the intensity $I(t)$ is determined by the values of ω_0, C_{cb}, V, $L(\omega_0)$ and $R(\omega_0)$. As mentioned above, the capacitance C_{cb} and the voltage V are given data, the inductance $L(\omega_0)$ and the resistance $R(\omega_0)$ are obtained with the help of (3) and (4), but, on the other hand, the value of ω_0 is unknown. This frequency is univocally related with the capacitance C_{cb} for a given set of coil and workpiece. The frequency ω_0 is the solution of the equation

$$\frac{4L(\omega)}{4\omega^2 L(\omega)^2 + R(\omega)^2} - C_{cb} = C(\omega) - C_{cb} = 0, \tag{9}$$

where the relation of the left hand side comes from reordering expression (6). Once ω_0 is known, we can evaluate $L(\omega_0)$ and $R(\omega_0)$ and calculate $I(t)$ with (5). In practice, we obtain the capacitance $C(\omega)$ for every frequency after computing $L(\omega)$ and $R(\omega)$ with (3) and (4). Then, when C_{cb} is fixed, we look for the capacitance C which accomplish $C(\omega) = C_{cb}$ to obtain ω_0.

Lorentz force on the workpiece

If we assume that the dimensions of the system coil-workpiece are small compared with the wavelength of the prescribed fields (quasi-static regime), the workpiece is linear, isotropic, homogeneous and non-magnetic and the modulus of the velocity at any point in the workpiece is always much less than 10^7 m/s then, the force per unit volume $\mathbf{f}(\mathbf{r},t)$ acting on the workpiece can be expressed as [3, 4, 5]

$$\mathbf{f}(\mathbf{r},t) = \mathbf{J}(\mathbf{r},t) \times \mathbf{B}(\mathbf{r},t) = \sigma\mu\left(\mathbf{E}(\mathbf{r},t) \times \mathbf{H}(\mathbf{r},t)\right), \tag{10}$$

where $\mathbf{r}$ is a point inside the workpiece, $\mathbf{J} = \sigma\mathbf{E}$ is the induced current density and $\mathbf{B} = \mu\mathbf{H}$ is the magnetic flux density. Under the above assumptions, the force acting on the workpiece can also be expressed as a magnetic pressure applied on its surface

$$P(\mathbf{r}_0,t) = \frac{1}{2}\,\mu\left(|\mathbf{H}(\mathbf{r}_0,t)|^2 - |\mathbf{H}(\mathbf{r}_\tau,t)|^2\right), \tag{11}$$

where $\mathbf{r}_0$ is a point placed on the workpiece surface nearest to the coil and $\mathbf{r}_\tau$ is a point placed on the opposite side.

Assuming quasi-static regime and linear materials, the field $\mathbf{H}(\mathbf{r},t)$ of (10) and (11) is related with the field $\mathbf{H}(\mathbf{r},\omega)$ calculated with the time-harmonic Maxwell's equations by means of the inverse Fourier transform

$$\mathbf{H}(\mathbf{r},t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathbf{H}_n(\mathbf{r},\omega)\, I(\omega)\, e^{i\omega t}\, d\omega, \tag{12}$$

where $i = \sqrt{-1}$ is the imaginary unit, $\mathbf{H}_n(\mathbf{r},\omega)$ is the magnetic field per unit intensity at the frequency ω and $I(\omega)$ is the Fourier transform of the intensity $I(t)$ flowing through the coil (equation (5))

$$I(\omega) = \frac{1}{2}\left(\frac{1}{\omega + \omega_0 - i\gamma} - \frac{1}{\omega - \omega_0 - i\gamma}\right), \tag{13}$$

being ω_0 the frequency given in equation (6) and γ the constant defined in (7). The magnetic field per unit intensity $\mathbf{H}_n(\mathbf{r},\omega)$ is defined by

$$\mathbf{H}_n(\mathbf{r},\omega) = \frac{\mathbf{H}(\mathbf{r},\omega)}{I_n}, \tag{14}$$

being $\mathbf{H}(\mathbf{r},\omega)$ and I_n the magnetic field and intensity appearing in (3). To relate $\mathbf{E}(\mathbf{r},t)$ with $\mathbf{E}(\mathbf{r},\omega)$ we make an equivalent development to the one shown above, being

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathbf{E}_n(\mathbf{r},\omega)\, I(\omega)\, e^{i\omega t}\, d\omega. \tag{15}$$

Optimum frequency and capacitance

The frequency at which the discharge current oscillates is a key parameter in the design of an electromagnetic forming system. In [6, 7, 8] it is shown that, for a fixed energy U and a given set of capacitor bank, coil and workpiece, there exist a frequency at which the maximum deformation of the workpiece is achieved. The use of this optimum frequency saves energy and prevents the premature wearing of the coil.

To modify the frequency of the discharge in an EMF system we change the capacitance C_{cb} of the capacitor bank. The relationship between C_{cb} and ω is described in equation (9). This relation is biunivocal and, therefore, the search for the optimum frequency ω_{op} is equivalent to the search for the optimum capacitance C_{op}. To obtain C_{op} we can compute the deformation of the workpiece for all the available values of C_{cb}. Thereafter, we select the C_{cb} which produces the maximum deformation. This is the approach followed in [6, 7, 8].

To reduce the number of simulations we can make an initial estimation C_{ie} and calculate the deformation produced by the capacitances C_{cb} with a value around C_{ie}. In fact, if C_{ie} is close enough to C_{op}, we can find C_{op} with only three coupled electro-mechanical simulations. To obtain C_{ie}, or equivalently ω_{ie}, we look for the frequency which produces the maximum momentum $\mathbf{P}$ in the first n semi-periods, where a semi-period is half a period $T/2 = \pi/\omega$. That is, we look for the frequency which makes maximum the quantity

$$\Delta \mathbf{P}_n = \int_0^{\frac{n\pi}{\omega}} \mathbf{F}_{tot} \cdot dt, \tag{16}$$

where $\mathbf{F}_{tot}$ is the total magnetic force acting on the workpiece and $\Delta\mathbf{P}_n$ is the momentum produced by this force in the first n semi-periods. The number n must be selected with care, if we use a high n then the frequency ω_{ie} will be too high and if n is too low then ω_{ie} will be too low. The number of semi-periods we select is the minimum natural number n which accomplish

$$n \geq \frac{2L\omega_\infty}{\pi R}, \tag{17}$$

where ω_∞ is the frequency which makes maximum the quantity $\Delta\mathbf{P}_\infty$ and L and R are the inductance and the resistance evaluated at ω_∞. The quantity $\Delta\mathbf{P}_\infty$ is obtained when $n \to \infty$ in (16). The expression (17) comes from reordering $(n\pi/\omega) \geq (1/\gamma)$, where γ is defined in (7). The minimum natural number which satisfies the inequality (17) represents the minimum number of semi-periods required to release more than 80% of the total momentum $\Delta\mathbf{P}_\infty$.

In summary, we first locate the frequency ω_∞ which makes maximum the quantity $\Delta\mathbf{P}_\infty$. Second, we compute n with (17). Finally, ω_{ie} is the frequency which makes maximum $\Delta\mathbf{P}_n$, being n the natural number calculated in the second step. All this process is performed neglecting the workpiece deformation. We do not require any additional simulation. We are using the data obtained in the initial frequency sweep of our electromagnetic model. It takes only a few seconds to compute all the integrals and obtain ω_{ie}.

APPLICATION EXAMPLE

In this section we apply the electromagnetic model explained above to the EMF process presented in [7]. This process consists in the expansion of a cylindrical tube by a solenoidal coil. In [7] is analyzed the tube bulging process under different working conditions, here, we will find the magnetic pressure acting on the workpiece when the capacitance is $C_{cb} = 160\,\mu$F and $C_{cb} = 800\,\mu$F, the coil length is $\ell = 200\,$mm and the initial charging energy is $U = 2\,$kJ. Also, we will compute C_{ie} for all the cases analyzed in [7].

The coil is made of $d = 2.0\,$mm diameter copper wire. The outer diameter of the coil is $D_c = 37.0\,$mm. The separation between each loop is $p = 3.0\,$mm. To compute the magnetic pressure we used a coil of length $\ell = 200\,$mm ($N = 68$ turns). We assume an electrical conductivity for copper of $\sigma = 58e6\,$S/m. The coil is approximated by coaxial loop currents, concentric with the workpiece and placed inside it. Thus, the problem is considered axis symmetric. The workpiece is a cylindrical tube made of annealed aluminum A1050TD with an outer diameter $D_{wp} = 40.0\,$mm and a thickness of $\tau = 1.0\,$mm. We assume an electrical conductivity for the workpiece of $\sigma = 36e6\,$S/m. We also assume that $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$ for workpiece and coil.

In [7] is said that L_{cb} and R_{cb} are less than $1.0\,\mu$H and $2.0\,$mΩ respectively. But, in [5], where it is used an EMF set-up similar to that used in [7], it is reached to the conclusion that these quantities do not take into account the inductance L_{con} and the resistance R_{con} of the wires connecting the capacitor bank with the coil. In [5] is found that $L_{cb} + L_{con} = 2.5\,\mu$H and $R_{cb} + R_{con} = 15.0\,m\Omega$ are more realistic values for the electrical parameters external to the system coil-workpiece. Then, in this work, we consider $R(\omega) = R_{cw}(\omega) + 15.0\,m\Omega$ and $L(\omega) = L_{cw}(\omega) + 2.5\,\mu$H as the total resistance and inductance of the RLC circuit formed by the capacitor bank and the system coil-workpiece.

We computed the fields $\mathbf{E}(\mathbf{r},\omega)$ and $\mathbf{H}(\mathbf{r},\omega)$ with the finite element tool ERMES. The problem was driven by a current density uniformly distributed in the volume of the coil wire. Then, inside the volume of the coil, equation (4) was replaced by

$$R_{cw}(\omega) = \frac{1}{|I_n|^2} \int_\upsilon \frac{|\mathbf{J}|^2}{\sigma}\, d\upsilon \tag{18}$$

where $\mathbf{J}$ is the imposed current density and σ is the conductivity of the coil. The values of $\mathbf{H}_n$, L, R and C obtained with ERMES as a function of the frequency $f = \omega/2\pi$ are shown in figures 1 and 2.

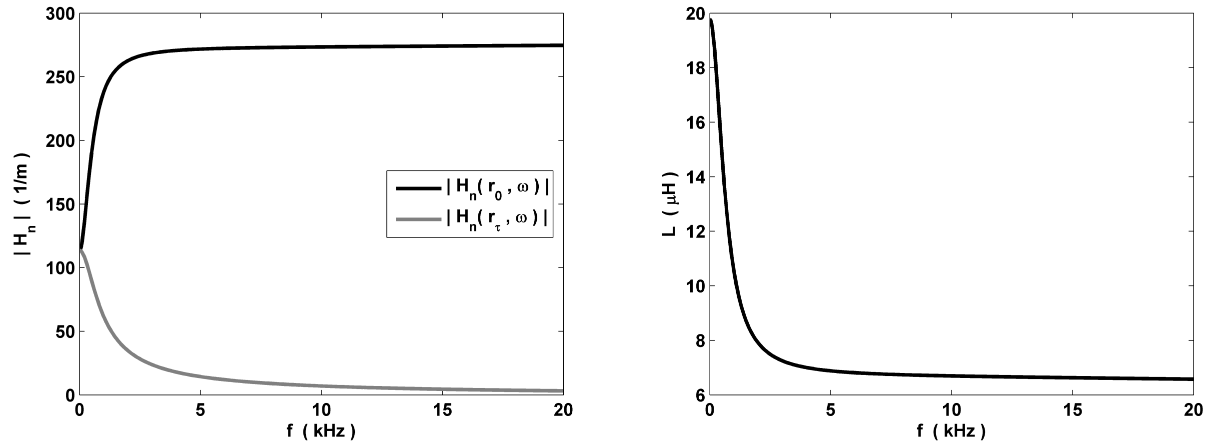

Figure 1. Left: Modulus of the normalized magnetic field on the workpiece surface nearest to the coil $|\mathbf{H}_n(\mathbf{r}_0,\omega)|$ and on the opposite side $|\mathbf{H}_n(\mathbf{r}_\tau,\omega)|$ as a function of the frequency $f = \omega/2\pi$. Right: Total inductance L of the RLC circuit formed by the capacitor bank and the system coil-workpiece as a function of the frequency f.

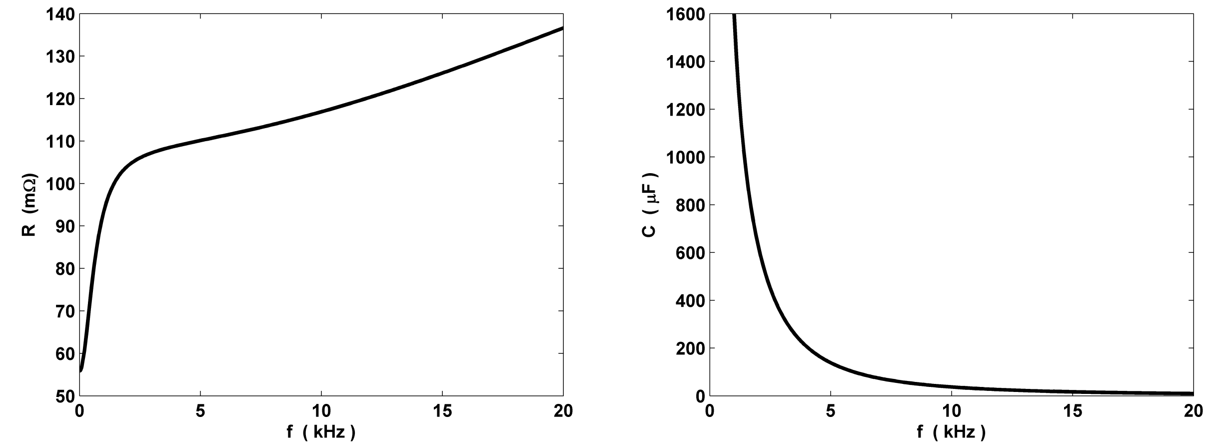

Figure 2. Left: Total resistance R of the RLC circuit formed by the capacitor bank and the system coil-workpiece as a function of the frequency f. Right: Capacitance C of the RLC circuit as a function of the frequency f.

We computed the intensity flowing trough the coil of length $\ell = 200\,\text{mm}$ ($N = 68$ turns) for the capacitances $C_{cb} = 160\,\mu\text{F}$ and $C_{cb} = 800\,\mu\text{F}$, being the initial charging energy of $U = 2\,\text{kJ}$ (see right graph of Fig. 3). For the case of $C_{cb} = 160\,\mu\text{F}$ we found that $f_0 = \omega_0/2\pi = 4.6\,\text{kHz}$, $L(\omega_0) = 6.9\,\mu\text{H}$ and $R(\omega_0) = 110.1\,\text{m}\Omega$. For the case of $C_{cb} = 800\,\mu\text{F}$ we found that $f_0 = 1.7\,\text{kHz}$, $L(\omega_0) = 8.3\,\mu\text{H}$ and $R(\omega_0) = 102.8\,\text{m}\Omega$. To obtain these values we only need to find f_0 in the right graph of Fig. 2 and use this frequency in $L(\omega)$ and $R(\omega)$. In Fig. 3 is also shown the maximum current intensities measured in [7] and the maximum current intensities computed in this work for several capacitances C_{cb} and initial charging energy $U = 1\,\text{kJ}$.

Once the intensity was known we used equations (13), (12) and (11) to compute the magnetic pressure acting on the workpiece. In Fig. 4 we show the magnetic pressure calculated in this work compared with the magnetic pressure calculated in [7]. We had to average the magnetic pressure over the surfaces of the workpiece to compare our results with those provided by [7]. The differences shown in Fig. 4 can be attributed to the features of each electromagnetic model. In [7] is considered the movement of the workpiece, assumed a uniform current density along all the the length of the coil and an exponential decay of the magnetic field from the inner surface to the outer surface of the metallic

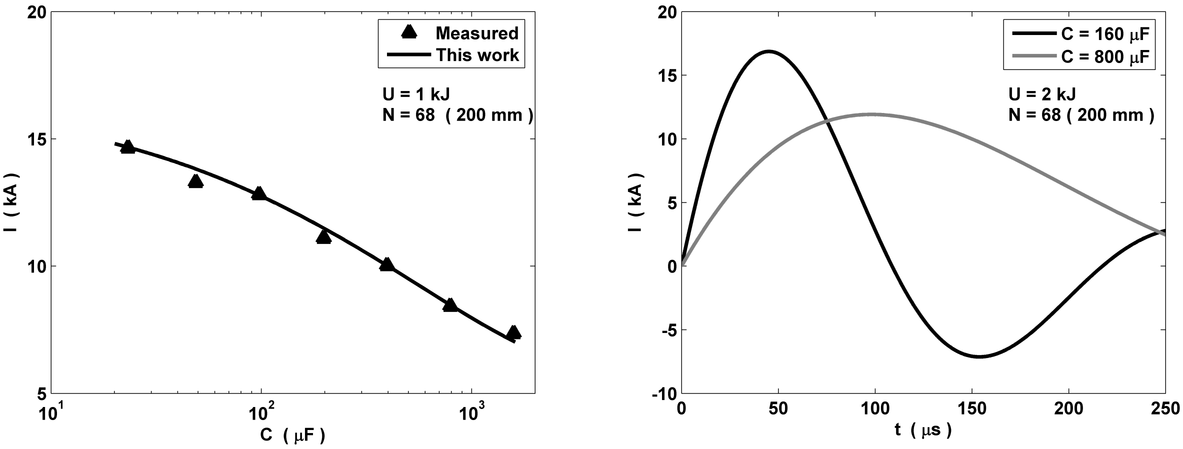

Figure 3. Left: Maximum current intensities measured in [7] compared with the maximum current intensities computed in this work for several capacitances C_{cb}. The initial charging energy is $U = 1\,\mathrm{kJ}$. Right: Intensity flowing trough the coil of length $\ell = 200\,\mathrm{mm}$ for the capacitances $C_{cb} = 160\,\mu\mathrm{F}$ and $C_{cb} = 800\,\mu\mathrm{F}$, being the initial charging energy $U = 2\,\mathrm{kJ}$.

tube. On the other hand, we neglected the workpiece deformation, considered a more realistic coil geometry and calculated numerically the magnetic field on the surfaces of the metallic tube.

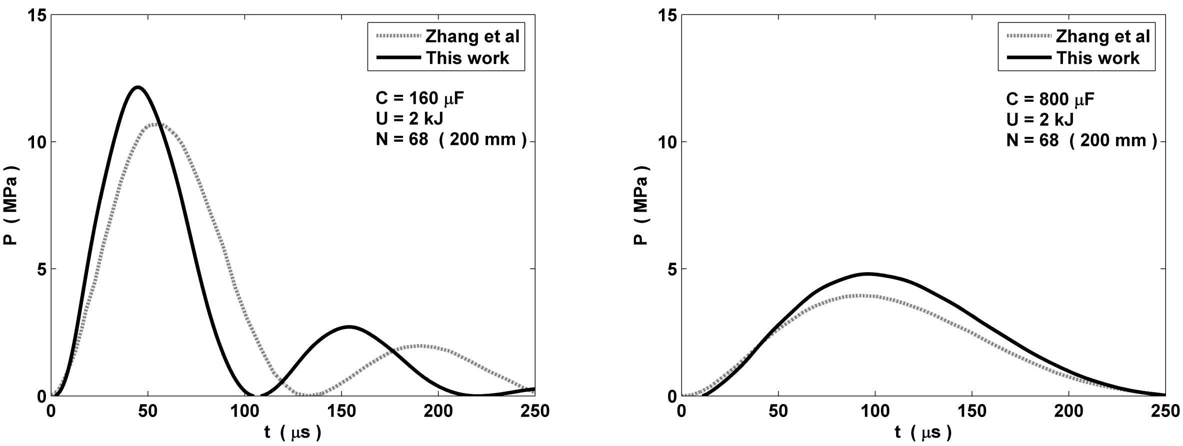

Figure 4. Magnetic pressure on the workpiece for the $\ell = 200\,\mathrm{mm}$ coil and initial charging energy $U = 2\,\mathrm{kJ}$. The magnetic pressure calculated in [7] is compared with the magnetic pressure calculated in this work. Left: $C_{cb} = 160\,\mu\mathrm{F}$. Right: $C_{cb} = 800\,\mu\mathrm{F}$.

In [7] is analyzed the expansion of a tube by a solenoidal coil under different working conditions. They used several coils with lengths $\ell = \{100, 200, 300, 400, 500\}\,\mathrm{mm}$. They computed for each coil the bulge height with the capacitance varying from $C_{cb} = 20\,\mu\mathrm{F}$ to $C_{cb} = 1600\,\mu\mathrm{F}$. They obtained the optimum capacitance C_{op} for each coil length and concluded that the optimum frequency satisfy $\delta = 0.9\,\tau$ in all the cases. We analyzed the same problem with the method expalined in the previous section. The number of semi-periods (17) was $n = 2$ in all the cases. Then, the initial estimation C_{ie} is the capacitance which makes maximum the quantity $\Delta\mathbf{P}_2$ defined in (16). In Fig. 5 we show the momentum $\Delta\mathbf{P}_2$ as a function of the capacitance C_{cb} for the coil lengths $\ell = 200\,\mathrm{mm}$ and $\ell = 500\,\mathrm{mm}$. The initial charging energy in both cases is $U = 2\,\mathrm{kJ}$. In Table 1 we show the values of C_{ie} obtained in this work compared with the optimum capacitances C_{op} calculated in [7]. In [7] is also analyzed a coil with $\ell = 100\,\mathrm{mm}$ and $U = 1\,\mathrm{kJ}$. They measure the bulge height for the capacitances $C_{cb} = \{24, 50, 100, 200, 400, 800, 1600\}\,\mu\mathrm{F}$ and they found that the maximum height was in $C_{cb} = 200\,\mu\mathrm{F}$. They also calculated numerically the optimum capacitance and they obtained a value of $C_{op} = 310\,\mu\mathrm{F}$. We calculated the initial estimation and we found that $C_{ie} = 296\,\mu\mathrm{F}$.

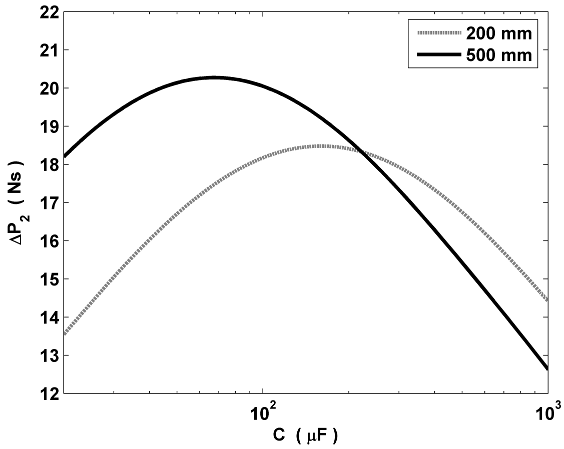

Figure 5. Momentum $\Delta\mathbf{P}_2$ as a function of the capacitance C_{cb} for the coil lengths $\ell = 200\,\text{mm}$ and $\ell = 500\,\text{mm}$. The initial charging energy is $U = 2\,\text{kJ}$. The initial estimation C_{ie} is the capacitance which makes maximum the quantity $\Delta\mathbf{P}_2$. The values of C_{ie} are shown in Table 1.

Table 1. Optimum capacitance (C_{op}) calculated in [7] and initial estimation (C_{ie}) calculated in this work for different coil lengths (ℓ)

$\ell\,$(mm)	100	200	300	400	500
$C_{op}\,(\mu\text{F})$	310	160	100	70	40
$C_{ie}\,(\mu\text{F})$	296	161	108	83	67

CONCLUSION

In this paper we have presented a numerical model for the simulation of electromagnetic forming processes. This method is computationally efficient because it only requires to solve the time-harmonic Maxwell equations for a few frequencies to have completely characterized an EMF system. The approach can be very useful for estimating the order of magnitude of some parameters, for experimentation on modeling conditions, for coil design or for modeling complex geometries. Moreover, it can be easily included in a sequential coupling strategy without the worry of numerical instabilities. The method also allows us to estimate the optimum frequency and capacitance at which it is attained the maximum workpiece deformation and it offers a new insight into the physics of EMF.

ACKNOWLEDGMENTS

This work is part of the project SICEM (SImulación multifísica para el diseño de Conformado ElectroMagnético) founded by the Spanish Ministry of Education and Science through the National R+D Plan 2004-2007, reference DPI2006-15677-C02-01.

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